JOURNAL OF AIRCRAFT
Vol. 29, No. 4, July–Aug. 1992

Structural Risk Assessment in the Israel Air Force for Fleet Management

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As operational aircraft accumulate service hours in the Israel Air Force (IAF), cracks begin to appear in structural components. IAF Logistic Center is often requested to determine whether flying can be continued without repair and the associated risk of failure. Those questions and many others cannot be addressed using only the Damage Tolerance Assessment. A computer program, "RISK," based on statistical methods, was developed in the IAF to provide fleet managers with quantitive assessments of the risk associated with each maintenance decision. This paper presents the theoretical background, details of the computer program "RISK" and samples of two risk analyses performed by the IAF using it.

Introduction

IN an effort to protect the structure of aircraft from catastrophic failure, the Israel Air Force (IAF) has adopted a deterministic damage tolerance design policy. This maintenance-directed program is based on a crack growth methodology and a tracking system that monitors aircraft usage data, aircraft inspection times, detected flaw sizes, and repair data. Almost every aircraft is equipped with an individual aircraft tracking (IAT) system.

The experience of the IAF with numerous damage tolerance assessments (DTA) analyzed by this method has shown that components which were not supposed to be in fatigue-critical locations sometimes fail in service. In these cases the Logistic Center is asked to determine the risk of subsequent failures, whether flying can be continued without repair, and which aircraft must be grounded and repaired. Experience has also shown that the predicted life of many fatigue-critical locations is much too conservative. The IAF Logistic Center is requested to determine whether the first inspection can be delayed and the risk associated with this approach.

Risk assessment provides the opportunity to address these and many other questions, such as "How many fatigue-critical components should fail in the next x years?", and "when a series of structural failures occur in a fleet, is there one or more modes of failure?". It should be emphasized that risk assessment is extremely important for fleet management purposes, and is complementary to the DTA.

It is the purpose of this paper to present the implementation of structural risk assessment for the benefit of fleet management in the IAF. A computer program, "RISK," was developed based on a statistical method that utilizes risk assessment methodology.

Theoretical background,[1,2] details of the computer code, and samples of risk analyses performed by the IAF are presented.

Theoretical Background

Risk assessment combines deterministic methods (DTA) and statistical methods to predict probability of structural failures. The approach is presented in Refs. 1 and 2. Using the IAT system and its large database, one can predict times-to-failure for cracked components, which are combined with measured lives for failed components to produce the combined failure data set.

A Weibull distribution is used to fit the combined failure data set for the following reasons: 1) it has broad applicability, 2) it provides a simple graphic solution, 3) it can be useful even when data is insufficient (e.g., sample size is small), and 4) it usually provides the best fit for the type of data encountered in structural fatigue failure.[3]

The form for the three-parameter Weibull probability distribution function for no failure is as follows:

$$F_{cr}(t) = 1 - \exp\{[(t - t_{0cr})/(\eta_{cr} - t_{0cr})]^{\beta_{cr}}\} \tag{1}$$

where t_{0cr} is the minimum expected failure time, η_{cr} is the characteristic failure time, and β_{cr} is the Weibull slope parameter for failure life.

The median cumulative distribution function associated with a ranked failure was used in the computations because it is independent of any distribution.[4] An excellent approximation for the median cumulative function can be shown to be

$$F(t_i) = (i - 3)/(N + 0.4) \tag{2}$$

where i is the ith ranked failure, N is the total number of failures in data set and t_i is the ith ranked failure life.

First, the two parameters for the Weibull distribution, β_{cr} and η_{cr} are determined (assuming that the third parameter $t_{0cr} = 0$) using two methods. β_{cr} and η_{cr} are estimated first by median rank regression (MRR), and then by the method of maximum likelihood estimators[5] using MRR estimation of the two parameters.[3]

Some components fail by causes other than the fatigue failure mode of interest (for example, static failure or corrosion failure). A component that failed by a different mode cannot be plotted on the same Weibull plot in the same manner as a component which fails due to fatigue, because the components do not belong to the same failure distribution. These data products are referred to as suspended points. The argument for including those suspensions in the analysis and their influence on fatigue failure rank order is described in detail in Ref. 3.

Safety considerations in aerospace operations require corrective action based on a very small sample of failure data. Since this is unusual compared to other Weibull applications, Monte Carlo simulation was used to study the accuracy of Weibull analysis when applied to data from a fleet of several thousand successfully operating components and very few (3–10) failures.[3] Both rank regression and maximum likelihood tend to overestimate β's with small failure samples (the slope on the Weibull plot is too steep). This positive bias

Received Jan. 22, 1991; revision received June 10, 1991; accepted for publication June 10, 1991.

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decreases as the number of failures increases. η is typically underestimated. Rank regression risk forecasts are conservative (overestimate the risk) and less precise when computed with few failures and many suspensions. Maximum likelihood risk forecasts are more accurate and precise than rank regression risk forecasts with small failure samples.

The third parameter for the Weibull distribution, t_{0cr}, is determined by an iterative method using β_{cr} and η_{cr}. Once β_{cr} and η_{cr} are determined (assuming $t_{0cr} = 0$), we can compute t_{0cr}

$$t_{0cr} = t_2 - \frac{(t_3 - t_2)*(t_2 - t_1)}{(t_3 - t_2) - (t_2 - t_1)} \tag{3}$$

where t_1 is the first failure time, t_3 is the last failure time, and t_2 is the time corresponding to the linear halfway distance on the Weibull plot vertical axis calculated by

$$t_2 = \exp\{w + \beta_{cr} * \ln(\eta_{cr})\}$$

where

$$w = y_1 + \tfrac{1}{2}*(y_3 - y_1)$$

$$y_1 = \ln\{\ln[1/(1 - F(t_1))]\}$$

$$y_3 = \ln\{\ln[1/(1 - F(t_3))]\}$$

After subtracting t_{0cr} from each failure time of the combined failure data set, the two parameters of the Weibull distribution (β_{cr} and η_{cr}) are recalculated using the "new" combined failure data set. t_{0cr} is recomputed using recalculated β_{cr} and η_{cr}. Those iterations are repeated until convergence of t_{0cr} is achieved.

A Weibull distribution is also used to characterize the current aircraft lives. The probability of failure of an aircraft component within the fleet after the next δt flying hours (assuming that the whole fleet flies the same number of hours) is determined by the following integral expression[2]:

Probability of failure after the next δt hours

$$= \int_0^\infty \exp(-x)*\exp[-\{[(\eta_{cr} - t_{0cr})*x^{1/\beta_{cr}} + t_{0cr} - t_{0pl} - \delta t]/(\eta_{pl} - t_{0pl})\}^{\beta_{pl}}]\,dx \tag{4}$$

where t_{0pl} is the minimum expected actual aircraft flight hours, η_{pl} is the characteristic flight hours, and β_{pl} is the Weibull slope parameter for actual aircraft flight hours.

It should be noted that the integral expression in Eq. (4) presents the probability of failure after the next δt hours, as long as

$$t_{0cr} \geq t_{0pl} + \delta t \tag{5}$$

If the whole fleet does not fly the same number of hours, then historical data can be used to estimate the number of flight hours of each aircraft in the next z years. A Weibull distribution will be used to characterize the aircrafts' expected lives for the next z years.

The Weibull distribution was selected because this method does not require that the type of distribution (normal, binormal, Poisson, etc.) be known in order to determine the probability density function. It will fit data that are not well fit by more common distributions.

The integral expression in Eq. (4) will then be

Probability of failure after the next z years

$$= \int_0^\infty \exp(-x)*\exp[-\{[(\eta_{cr} - t_{0cr})*x^{1/\beta_{cr}} + t_{0cr} - t_{0pl}(z)]/[\eta_{pl}(z) - t_{0pl}(z)]\}^{\beta_{pl}(z)}]\,dx \tag{6}$$

where $\beta_{pl}(z)$, $\eta_{pl}(z)$, $t_{0pl}(z)$ are the three Weibull parameters for expected flight hours for the next z years.

The integral expressions Eq. (4) and Eq. (6) are computed numerically using a fifteen point Laguerre-Gauss quadrature[6,7]

$$\int_0^\infty \exp(-x)*f(x)\,dx = \sum_{i=1}^{m} H_i*f(x_i) + \text{Discretization error} \tag{7}$$

where x_i is the ith zero of mth Laguerre polynomial, and H_i is the ith weight factor, while $m = 1, \ldots, 15$.

Based on several structural risk analyses, it was found that $f(x)$ approaches zero very quickly ($f(x_i) \approx 0.0$ for $x_i \geq 1$), making only the first two terms in the above sum significant. The accuracy of the numerical integration is being controlled by computing the error between two consecutive sums, e.g.:

$$\sum_{i=1}^{m} H_i*f(x_i) - \sum_{i=1}^{m-1} H_i*f(x_i)$$

Once the probability of failure is determined as a function of δt [Eq. (4)] or as a function of years [Eq. (6)], the number of components expected to fail at any time can be determined.

For a large, complete (no suspensions) failure data set of size N, the confidence intervals for β_{cr} and η_{cr} can be approximated by the following expressions[3]:

$$\beta_{cr}*(-0.78*Z_\alpha/\sqrt{N}) \leq \beta_{cr} \leq \beta_{cr}*(0.78*Z_\alpha/\sqrt{N}) \tag{8}$$

$$\eta_{cr}*(-1.05*Z_\alpha)/(\beta_{cr}*\sqrt{N}) \leq \eta_{cr} \leq \eta_{cr}*(1.05*Z_\alpha)/(\beta_{cr}*\sqrt{N}) \tag{9}$$

where Z_α, the upper $\alpha/2$ point of the standard normal distribution, depends on the confidence level chosen. Z_α values for various (usual) confidence levels are presented in many statistical handbooks.

The confidence bands are computed by expressions (4) or (6) using the values derived by inequalities (8) and (9). The confidence intervals are only approximate, since these estimates are only approximately normally distributed.

A failure data set may contain more than one mode of failure. In this case two distinct failure distributions will appear on the Weibull plot (see Fig. 1). To detect this phenomenon, only measured lives for failed components should be

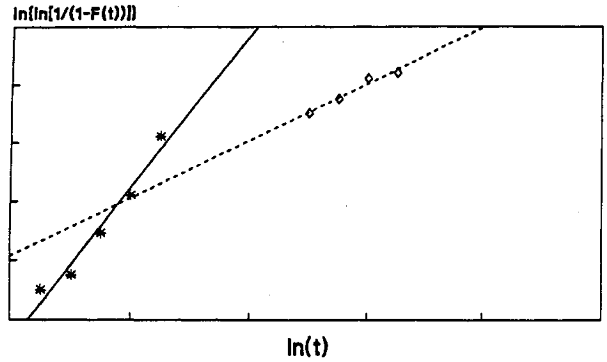

Fig. 1 Two distinct failure distributions on the Weibull plot.

incorporated into the failure data set. Risk analysis should be performed for each separate mode of failure.

Computer System ("RISK") Description

"RISK" is a Fortran program and is run on an IBM 3090 mainframe. Based on theoretical background, "RISK" can predict the following: the number of critical components that are expected to fail in the next z years; whether one or more modes of failure will appear; the risk associated with the delay of an inspection; and the risk of subsequent failure when a series of structural failures occur in the fleet.

Each type of aircraft that has an IAT system can be interfaced to the "RISK" program (Fig. 2), for the purpose of preparing the combined failure data set as described in Refs. 1 and 2, and preparing the current component lives.

The input requirements include: combined failure data set, current component lives, number of flight hours and fleet is supposed to fly in the next z years, numerical error limit desired for numerical integration convergence, numerical error limit desired for t_0, and reporting requirements.

"RISK" output consists of tables, graphs, and numerical data on the following: 1) the three Weibull parameters for failure and current component lives data sets (after median rank regression and after maximum likelihood estimations); 2) graphic representation of the fit of the failures and current component lives to estimated Weibull distributions (see Fig. 3); 3) graphic representation of estimated Weibull distribution (see Fig. 4); 4) rate of convergence of numerical integration and estimated numerical error; and 5) graphic representation of the number of components supposed to fail as a function of fleet flight hours, including upper and lower range confidence levels (see Fig. 5).

Special attention is paid to a number of factors when the results are analyzed. The graphic representation should clarify whether the assumption of only one failure mode is correct. The distribution of data points around the line estimated as best fit on the Weibull plot must be closely examined. A large amount of scatter or arrangement along a nonlinear curve indicates the Weibull distribution may not be the correct one. The numerical integral convergence must also be examined, and the numerical error should not exceed the requested value. When $f(x)$ [see Eq. (7)] approaches zero very quickly, the numerical error is usually relatively large. Finally, confidence levels for η_{cr} and β_{cr} that are too large preclude the drawing of significant conclusions.

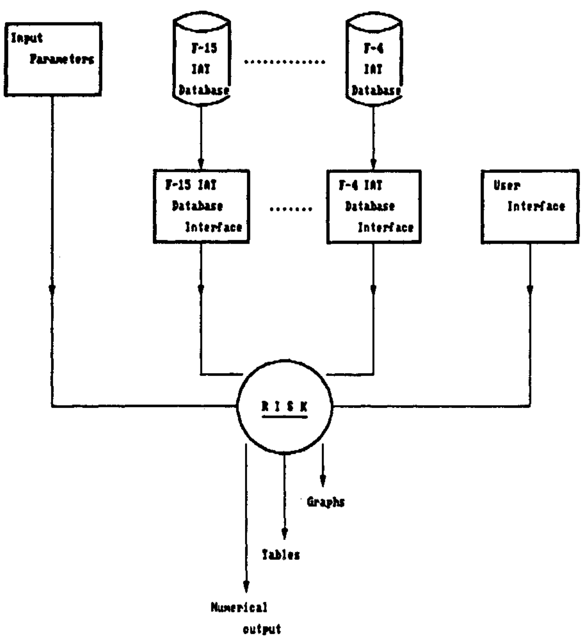

Fig. 2 Diagram of "RISK"-IAT system interface.

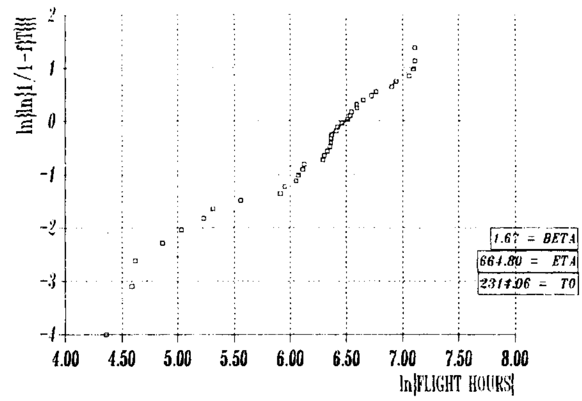

Fig. 3 Weibull plot for failure.

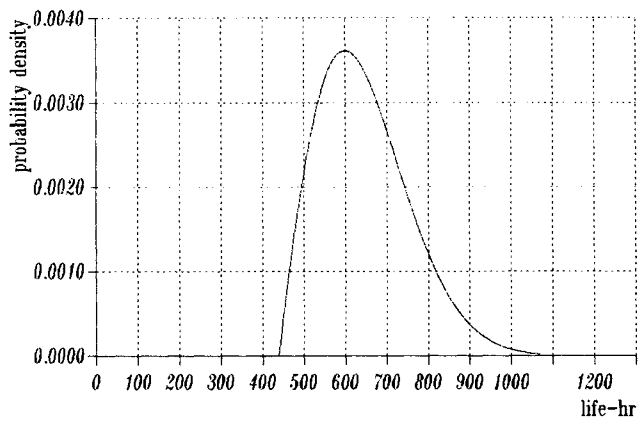

Fig. 4 Weibull probability density function: flight hours.

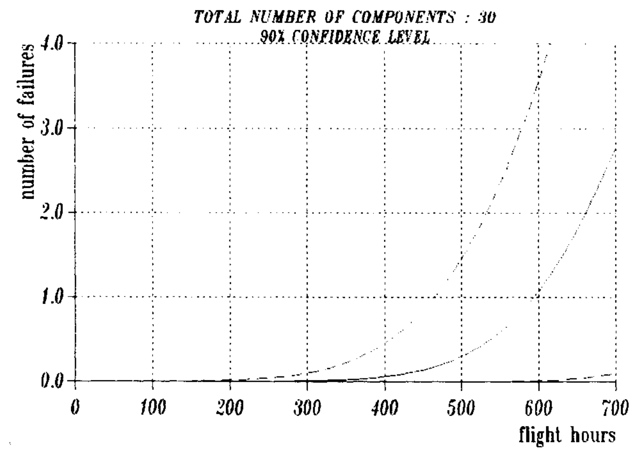

Fig. 5 Failures of components vs flight hours.

Efforts are underway in the IAF to improve the system. New, more precise methods are being applied to numerical integration for cases where $f(x)$ presented in Eq. (7) approaches zero very fast.

Weibayes is defined as Weibull analysis with an assumed β parameter.[3] In a Weibayes analysis, the slope/shape parameter β is assumed from historical failure data or from engineering knowledge of the physics of the failure. Depending upon the situation, that may be a strong or weak assumption. Weibayes analysis is being developed to solve problems when a Weibull plot cannot be used because of limitations in the data. Typical situations are when there are too few or no failures, and when the age of the components is unknown and only the number of failures is known.

The current program does not account for the replacement of repair of failed/cracked components and their return to service. Analytic techniques that address this problem and can be integrated into the "RISK" program are being investigated.

The detection probability function for nondestructive inspections was assumed to be a "step function," which equals one for a crack size greater than a specific value, and zero otherwise. In reality, nondestructive inspection techniques have uncertainty associated with their ability to detect flaws. The shape of the probability of detection function for some nondestructive inspection techniques is available in Ref. 8. Uncertainty in inspection data results may be reflected in risk assessment by using a Monte Carlo technique as recommended in Ref. 9, or by other methods that may be incorporated into the IAT interface to "RISK" program.

Samples of Risk Analysis

Example A

Nondestructive inspections were implemented on 22 wing intermediate spars, upper flange, on an IAF fighter. The inspections, which were performed on the 11 "oldest" aircraft, detected flaws on each inspected aircraft. A typical figure of a crack is illustrated in Fig. 6. If cracks exceed the "repairable" length, the intermediate spar should be replaced, meaning significant downtime and very expensive repair. Crack sizes and crack propagation directions measured while structural repairs were implemented revealed no correlation between crack size and an aircraft's accumulated fatigue damage index, or between crack size and the number of flying hours.

Additional flaws were detected when the same structural region was inspected on thirty wings of "newer" aircraft. The IAF Logistic Center was asked to determine the time period flying could be continued without repair and the associated risk of a crack exceeding the "repairable" length. Based on the crack lengths determined on the "oldest" aircraft, and the accumulated flying hours of each aircraft, the life until a crack reaches an "unrepairable" length was calculated for the "oldest" aircraft as

$$\frac{\text{accumulated flying hours} * \text{maximum "repairable" crack length}}{\text{detected crack length}} \tag{10}$$

Theoretically, crack propagation time for the "newer" aircraft should be the same as for the "oldest" aircraft, since both fly with the same spectrum, and the spar material and geometry of both are identical. Lives until cracks reach "unrepairable" length, and the actual lives of "newer" aircraft were input to "RISK." Probability density functions fitted to both data sets are presented in Figs. 4 and 7. The number of expected wings on which "unrepairable" cracks might be detected vs fleet accumulated flight hours (with a 90% confidence level band) is presented in Fig. 5.

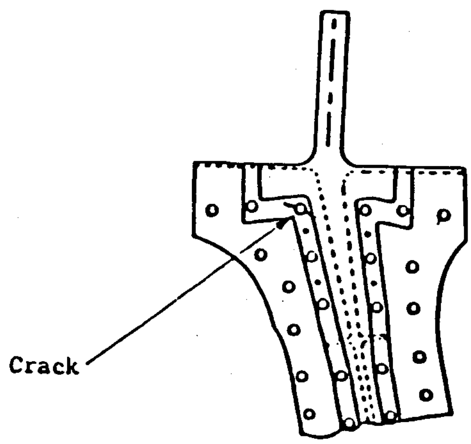

Fig. 6 Intermediate spar, upper flange.

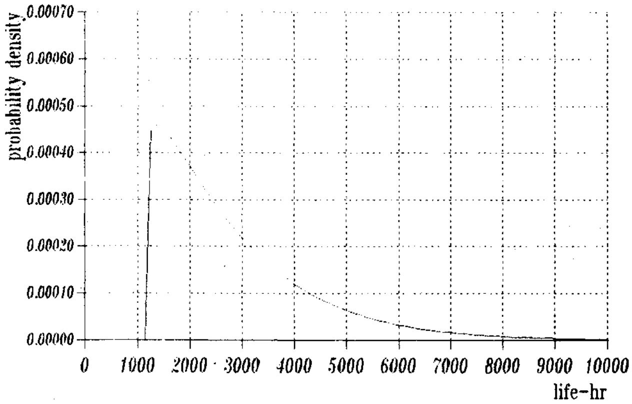

Fig. 7 Weibull probability density function: failure.

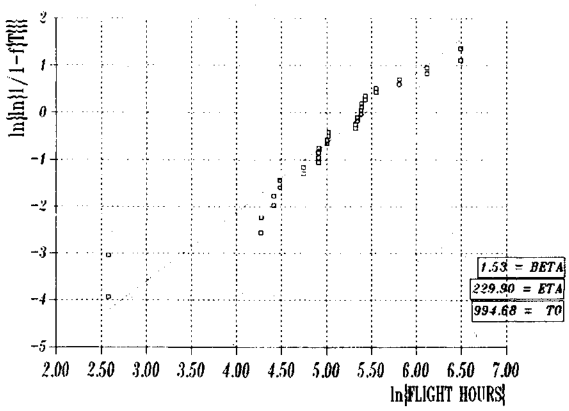

Fig. 8 Weibull plot for failure.

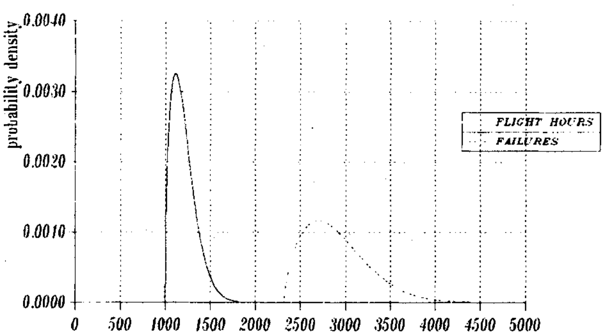

Fig. 9 Weibull probability density functions: flight hours and failures.

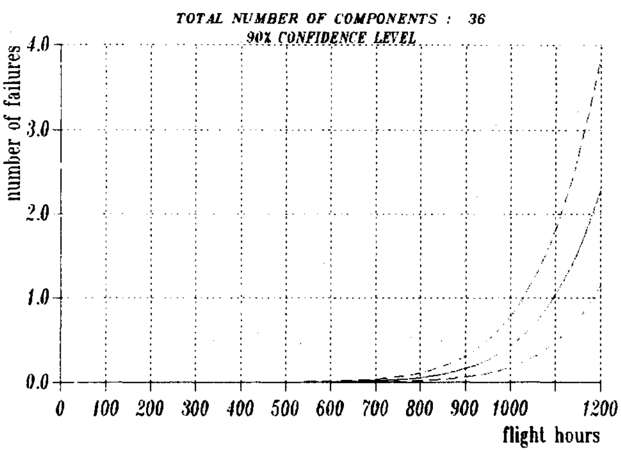

Fig. 10 Failures of components vs flight hours.

Risk analysis results recommend that all "newer" aircraft be repaired before the fleet accumulates 300 additional flight hours. The chance of detection of a crack which is "unrepairable" increases very rapidly after 400–600 fleet accumulated flight hours. As the "newer" aircraft are being repaired, more information on additional crack sizes will be available, enabling a more accurate risk analysis.

Example B

One of the most critical fatigue locations on same IAF fighters is the intermediate spar lower flange. According to the IAT system, this location should have been already inspected on several aircraft. However, lack of a nondestructive inspection procedure precluded performing the required inspection. Risk analysis was applied to predict the time period flying could be continued with extremely low risk without inspecting the structural detail.

Analytical time-to-failure for each individual aircraft was calculated based on individual aircraft usage and the normalized crack growth curve for the location, using the IAT system and IAT interface to "RISK." The actual fleet flight hours data set and the time-to-failure data set were input to "RISK."

Figures 3 and 8 present the distribution of data points—both failures and actual flight hours around the lines estimated as best fit on Weibull plot. Some amount of scatter exists around the estimated lines, indicating that the data sets do not match "perfectly" to a Weibull distribution; however, the data sets match relatively well to estimated lines.

Figure 9 presents the probability density function for the actual flight hours data set compared to the probability density function distribution for the failure data set. The number of expected failures as a function of fleet accumulated flight hours is presented in Fig. 10.

The risk of failure for the next 600–700 flying hours is extremely low, and flying can be continued relatively safely without inspections. However, when failures do begin to appear in the fleet many failures will occur in a very short period. Compared to the previous analysis, the confidence level band is narrower, allowing a more precise prediction of the number of expected failures with the 90% confidence level. The results may be changed because of a lack of detected flaws, scatter of data points around lines estimated as best fit on Weibull plots, and numerical errors arising from the numerical integration. Nevertheless, at the present stage, risk analysis seems to provide fleet managers with the best answers based on available information.

Conclusion

The structural engineer must use all methods available to ensure flight safety in aircraft with cracks due to fatigue. The "RISK" system, based on statistical methods, is now available to the IAF to provide fleet managers with quantitative assessments of the risk associated with each maintenance decision and to protect the safety of flight.

"RISK" capabilities are being used to insure better fleet readiness and reliability.

Acknowledgments

The author acknowledges the work of Itzhak Shakasta and Michael Kupferman and their contribution to this effort. The author thanks the management of the Structure Branch for the opportunity to work on this project.

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